Analysis Qualifying Exam

Part I: Complex Analysis

August, 2018

1. Prove that if f is entire and the imaginary part of f is bounded on $\mathbb{C}$, then f must be constant.

2. Find a conformal map from the unbounded region outside the disks $\{|z+1| \leq 1\} \cup \{|z-1| \leq 1\}$ to the upper half plane.

3. The function $\frac{z^3-1}{z^2+4z-5}$ has a power series expansion in a neighborhood of the origin. What is its radius of convergence?

4. Let γ be the closed polygon $[3-i, 3+i, 1/2+i, 1/2-i, 3-i]$. Find the following integrals

$$\int_\gamma z^m/(z-1)^m, \qquad m \in \mathbb{N}$$

5. Find the residue of

$$f(z) = \frac{e^{z^2}}{z^n}$$

at each of its pole in $\mathbb{C}$ for each $n \in \mathbb{N}$.

6. Evaluate

$$\int_0^\infty \frac{dx}{(x^2+4)^2}$$

7. Suppose $f : D \to D = \{z : |z| < 1\}$ analytic and $f(z)$ is not identically equal to z. Show that f can have at most one fixed point in D.